# Dynamic synthesis of non-harmonic vibration exciter of pulse action directed on the foundation

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Abstract
The problem of dynamic synthesis for non-harmonic vibration exciter of hinge-lever type is discussed in the current article. The article also reports on the developed mechanism for vibration exciter and its pulse action with regard to the foundation. The numerical results of the stated problem are described.
Key words: VIBRATION, FOSSILS, ECCENTRIC MASS, LEVER MECHANISM, PULSE, DYNAMIC SYNTHESIS, APPROXIMATION, MASS AND INERTIA PARAMETER, INERTIA

**Introduction**

The vibrating mechanisms are widely applied in various industrial fields and agriculture. They are of significant importance for operations in metallurgy. With vibration, people can crush materials, separate mixtures, compact concrete, sink piles, screen various substances in industry; the vibration phenomenon is also applied in the household. A number of manufacturing processes cannot produce without vibrating technology: extracting and processing of fossils and minerals, handling substances in chemical production, participation in the metallurgical route, building materials handling and erection of various engineering constructions [5, 6, 7]. All the machines involved in the above described processes contain various types of vibration exciters [7, 8].

The analysis on the research papers, devoted to the problem of vibration exciters and their development, reveals the issues to be improved. The point is that the most widely applied equipment here is inertial vibration exciter, in which the exciting force is generated by rotation of several unbalanced masses. The directional effect of the force exciting in it is provided by the vibration exciter of self-balancing type. This exciter consists of twin exciting units with eccentric masses, which synchronically rotate in opposite directions but with the same rotational speed. In order to synchronize the rotation of the eccentric masses, the kinetic pair is used. Such an arrangement of the mechanism imposes teeth wearing, but the engineering solution to apply separate drives is to raise the problem of the synchronic operation. Moreover, considering the operational conditions of self-balancing vibration exciter from the standpoint of the effect produced on the foundation, we observe that it influences the foundation harmonically and this type of equipment (for instance, pulse class) is not feasible to have non-harmonic impact on it. In order to generate mechanical vibrations of directional non-harmonic effect, the exciter with non-rounding are applied [4, 11]. However, the main rebukes on this mechanism are the teeth wear under impact and complicated shapes of teeth manufacture for this type of equipment.

The most common vibration exciters have non-harmonic impact on the foundation and are equipped with planetary gear [12, 13]. Among the principle drawbacks in their designs, we may single out the ones related to the kinetic pairs of higher degree in their constructions: the kinetic pair provokes elements wearing, generates the adverse vibrations transversely (there are certain difficulties with developing the directed impact on the foundation) and enables the drive trailing wheel slipping. Furthermore, due to the mentioned problems, this type of equipment requires special constructions.

The lever mechanism with its wide functionality is not applied in most cases here [4, 11]. However, the elimination of kinetic pair of higher degree in the above described mechanisms is able to ensure high speed modes of operation and to increase the parts service life. By choosing mass and inertia parameters or implementing rational arrangement of mass link centres and optimizing mass and inertia moment values, one can obtain actually any desired type of impact on the foundation and directed non-harmo- nic one, in particular, on condition of the appropriate regularity. However, nowadays there is no optimization mathematical technique of dynamic synthesis for the described machines and this makes a considerable obstacle for the developments in this direction. In addition to the above, the automating perspectives in design process are becoming important requirements to the modern techniques applied for machines and mechanisms construction.

**The present article aims at** development of the optimal method for dynamic synthesis of hinge-lever mechanisms based on the vibration exciter of the directed pulse effect on the foundation.

**1. Analytical solution for dynamic synthesis**

**1.2 Synthesis equations**

Let us assume that *OXY* is a system of absolute coordinates. Let us also regard that *OX* axis is directed along the line joining the centres of two hinges, this is done as a matter of convenience.

Consider the normalized conditions under which the mechanism produces impacts on the foundation with $f_\tau^0$, $f_n^0$ and acts in two orthogonal direction τ and n ($\tau^T n = 0$), at this the axis is rotated with angle $\psi_1$ with respect to *OX* axis [5]. Thus, making a solution on the synthesis of non-harmonic vibration exciter, which produces τ direction pulse impact on the foundation, the action is equal to zero:

$$f_\tau^0 \equiv 0, \quad f_n^0(\phi) = \begin{cases} A_1 \; at & 0 \le \phi < \phi_1 \\ A_2 \; at & \phi_1 \le \phi < 2\pi \end{cases}$$

Relation $\lambda = \dfrac{A_1}{A_2}$ is to be set within 3 ÷ 8. $Ml_1\omega^2$, where

$$M = \sum_{i=2}^{n} m_i,$$

$l_1$ is the distance between the supports, and we write as:

$$f = \frac{F}{Ml_1\omega^2} \tag{1}$$

where $f = [f_x, f_y]^T$ and $F = [F_x, F_y]^T$. Moreover, for the synthesis solution we accept that ω = const, ε = 0, then for any six link mechanism $(n=6)$ with the constituents of the principle vector of the inertia forces in normalized form, one can express as follows:

$$f_x = \sum_{i=2,4,6} \left[ A_i \left( \varphi_i'^2 \cos\varphi_i + \varphi_i'' \sin\varphi_i \right) + B_i \left( \varphi_i'' \cos\varphi_i - \varphi_i'^2 \sin\varphi_i \right) \right] \tag{2}$$

$$f_y = \sum_{i=2,4,6} \left[ B_i \left( \varphi_i'^2 \cos\varphi_i + \varphi_i'' \sin\varphi_i \right) - A_i \left( \varphi_i'' \cos\varphi_i - \varphi_i'^2 \sin\varphi_i \right) \right] \tag{3}$$

Furthermore, in the previously developed expressions for $A_i$, $B_i$, $i = 2, 4, 6$ all the linear dimensions are treated as relative (with respect to $l_1$ length or *AD* is taken as 1 unit), all the masses are non-dimensional and $\omega = 1$. As vectors, these expressions are written in the form as:

$$\mathbf{f} = \sum_{i=2,4,6} C(\phi_i', \phi_i'') D(\phi_i) \mathbf{u}_i \tag{4}$$

where

$$\mathbf{u}_i = [A_i, B_i]^T, \quad \mathbf{B}(\varphi', \varphi'') = \begin{bmatrix} \varphi'^2 & \varphi'' \\ -\varphi'' & \varphi'^2 \end{bmatrix}$$

As the preset directions (τ As the preset directions (**τ, n**) of foundation impact are rotated with angle $\psi_1$ with respect to *OXY* coordinate system, which is rigidly jointed with the stand, then we can develop the synthesis equation by rotation of the fixed coordinate system with the turn for the stand with angle $\psi = -\psi_1$ (this corresponds to the turn of the whole mechanism with the stand). Thus, the synthesis equation is as follows:

$$AU = f^0 \tag{5}$$

where $A = D(\psi)[C_2D_2 | C_4D_4 | C_6D_6]$, $\mathbf{U} = [\mathbf{u}_2^T, \mathbf{u}_4^T, \mathbf{u}_6^T]^T$, $\mathbf{f}^0 = [f_{\hat{o}}^0, f_n^0]^T$.

The dimensions of A matrix and U vector are dim A = 2x6 and dim U = 6 respectively.

It is important to emphasize that A matrix depends only on the geometrical dimension and linkage parameters (on the position, analogs of velocities and link accelerations). On the other hand, U vector includes all the mass and inertia parameters, in case of six link linkage, the total number of them is only 15 (the dimension of U vector is equal to 6); the components of U vector are generalized eccentric masses.

**1.3 Explicit solution for dynamic synthesis problem**

For the analytical synthesis of the vibration exciter, we accept that the mechanism kinematic analysis is carried out at the preset links dimensions and at the crank angular velocity ω = 1, (ε = 0). This results in determining the angular positions, angular velocities and acceleration of all the links in the preset N positions of crank 2. However, we do not need to know more than the values for the even numbered links. An additional point is that values $f_k^0 = [f_{\tau k}^0, f_{nk}^0]^T$, k=1,…,N are set for the problem of the dynamic synthesis. This corresponds to the preset N positions of the mechanism.

The equations for synthesis can be written as follows:

$$\boldsymbol{\delta}_k \equiv \mathbf{A}_k \mathbf{U} - \mathbf{f}_k^0 = 0,\ k=1,\ldots,N \quad (6)$$

Hence, we obtain $2N$ equations of synthesis, while the number of variables is 6. Therefore, the problem under discussion has the explicit solution provided that $N = 3$, in other words, we set numerical values for the component of total inertia force with three set position of the mechanism. In this case, in order to determine unknown parameters of **U**, we are to receive six linear equations with six variables.

$$\begin{bmatrix} [A_1] \\ [A_2] \\ [A_3] \end{bmatrix} \mathbf{U} = \begin{bmatrix} f_1^0 \\ f_2^0 \\ f_3^0 \end{bmatrix} \quad (7)$$

Eventually, the obtained equations have the only solution, on condition that the determinant *det* is not equal to 0:

$$\begin{bmatrix} [A_1] \\ [A_2] \\ [A_3] \end{bmatrix} \neq 0$$

**2.1 Quadratic approximation solution**

For the general case, if $N$>3, then the number of variables is bigger in equations (6) than the possible number of variables. Owing to this, the explicit solution is absent. We need to apply here approximate solution, where equations (6) are performed approximately [7].

$$S_2 = \|\Delta\|_2^2 = \frac{1}{2}\Delta^T W^T W \Delta \quad \Rightarrow \quad \min_U \quad (8)$$

where

$$\boldsymbol{\Delta} = \left[\boldsymbol{\delta}_1^T, \boldsymbol{\delta}_2^T, \ldots, \boldsymbol{\delta}_N^T\right]^T, \quad \boldsymbol{\delta}_k = \left[\delta_k^n, \delta_k^\tau\right]^T,$$

$$\mathbf{W} = diag\left\{w_1, w_2, \ldots, w_{2N}\right\}$$

that is the matrix of the weight numbers.

The solution problem (126) is written as follows:

$$U^0 = C_p^{-1} d \quad (9)$$

where

$$C_p^{-1} = \left[C^T W^T W C\right]^{-1} C^T W^T W$$

is the left pseudoinverse of matrix

$$\mathbf{C},\ \mathbf{C} = \left[\mathbf{A}_1^T, \mathbf{A}_2^T, \ldots, \mathbf{A}_N^T\right]^T,$$

$$d = \left[f_1^{0T}, f_2^{0T}, \ldots, f_N^{0T}\right]^T.$$

This solution can be found from the necessary conditions for the minimum of function $S_2$:

$$\frac{dS_2}{dU} = 0.$$

Furthermore, the matrix of second derivatives

$$\frac{dS_2}{dU} = C^T W^T W C$$

is nonnegative definite along with leading subdeterminants. Therefore, when *det* $C^T W^T W C \neq 0$, the necessary conditions for the minimum are the sufficient ones.

**2.2 Analytics and optimization method**

In the dynamic synthesis developed for the mechanism, its mass and inertia parameters are determined as total eccentric masses, and the mechanism dimensions are suggested to be preset. However, at preset geometric parameters, it is very difficult to obtain the desired approximation accuracy. This makes us to perform the mechanism synthesis by all **X** parameters at once, that is to search not only mass and inertia parameters for **U**, but also the mechanism metric parameters $\mathbf{X}_1 = \mathbf{X}/\mathbf{U}$ (links dimensions). These parameters are included into the effectiveness function (6) nonlinearly: kinematic parameters $\varphi_i, \varphi_i', \varphi_i''$, which participate in the approximation error expression, depend on them. Thus, in order to determine the rest of nonlinear parameters of the mechanism geometrical dimensions $\mathbf{X}_1 = \mathbf{X}/\mathbf{U}$, we introduced modified effectiveness function taking into account an analytical solution as follows:

$$S^m \equiv \min_{\mathbf{U}} S_2(X_1, U) = S_2\left(X_1, C_p^{-1}(X_1) d(X_1)\right) \quad \Rightarrow \quad \min_{X_1} \quad (10)$$

Therefore, the introduction of generalized eccentric masses and analytical solution for the mass and inertia variables allows a sufficient reduce in the dimensionality of the original optimization solution.

Based on the suggested method, the dynamic synthesis program for mechanisms with vibration exciters of hinge-lever type is developed. Mass and inertia parameters of the mechanism are determined on each step of the descent algorithm by minimization of the Euclidean or Chebyshev norms of approximation error. Numerical implementation of minimization procedure is carried out on the basis of Nelder Mead algorithm.

**2.3 Numerical results**

Summarizing the results of the researches made, we would like to report that there is a program developed, which provides a possibility for the user to denote optimization parameters and those ones which freely varied or scanned. The initial optimization parameters values and the scanned parameters values are defined with the use of the generator of LPt sequences, uniformly distributed in the searching range [9]. We have to note that the further nonlinear optimization is not applicable with the scanned parameters.

Moreover, the synthesis with Nelder Mead optimization method or the method of deformed polyhedron is carried out under the conditions described below.

The coordinates of stands have fixed values: $X_A = Y_A = 0.0$, $X_D = 1.0$, $Y_D = 0.0$, $X_G = Y_G = 0.0$.

The bounds of optimized variables are being changed under the following limits:

- $0.5 \le l_3 (BC) \le 1.4$
- $0.35 \le l_4 (CD) \le 1.2$
- $0° \le \theta_3$ (angle $CBE$) $\le 360°$
- $0.5 \le a_3 (BE) \le 1.2$
- $0.4 \le l_5 (EF) \le 1.1$
- $0.45 \le l_6 (FG) \le 1.1$
- $0° \le \psi \le 360°$
- $0° \le \varphi_{0cr} \le 360°$ (here $\varphi_{0cr}$ is the initial angle of crank turn)

Thus, optimization is carried out with 8 parameters. The generalized eccentric masses are determined on each step of the optimization, which makes numerical optimization significantly easier.

Parameter $l_2$ is the length of $AB$ crank and is not employed by optimization parameters: its values vary freely within $0.1 \le l_2 \le 0.4$ along with initial values of optimization parameters.

Furthermore, we chose the dyad groups of $BCD$ and $EFG$: $i_{BCD} = -1$, $i_{EFG} = -1$.

Table 1 shows the desirable values for component $\mathbf{f}^0$, in other words $f_n^0$ and $f_\tau^0$ in $N_S = 18$ positions of crank (the angles of $\varphi_{cr} f_i$ and $f_\tau^0$). The last column of the table contains the values of weight numbers $w_k$, $k = 1,\dots, N_S$. The proper choice made on these weight numbers determines the numerical results of optimization. At the first stage of the procedure, these values are assumed as equal to 1 unit element ($w_k = 1$).

**Table 1**. The desirable values for component $\mathbf{f}^0$

| $k$ | $\varphi_{cr}$ | $f_n^0$ | $f_\tau^0$ | $w$ |
|---|---|---|---|---|
| 1 | 0.00000 | 0.30000 | 0.00000 | 0.10000 |
| 2 | 15.00000 | 0.90000 | 0.00000 | 0.10000 |
| 3 | 30.00000 | 1.30000 | 0.00000 | 1.00000 |
| 4 | 45.00000 | 0.90000 | 0.00000 | 0.00000 |
| 5 | 60.00000 | 0.30000 | 0.00000 | 0.00000 |
| 6 | 80.00000 | -0.20000 | 0.00000 | 1.00000 |
| 7 | 100.00000 | -0.20000 | 0.00000 | 3.00000 |
| 8 | 120.00000 | -0.20000 | 0.00000 | 2.00000 |
| 9 | 160.00000 | -0.20000 | 0.00000 | 1.00000 |
| 10 | 180.00000 | -0.20000 | 0.00000 | 1.00000 |
| 11 | 200.00000 | -0.20000 | 0.00000 | 1.00000 |
| 12 | 220.00000 | -0.20000 | 0.00000 | 1.50000 |
| 13 | 240.00000 | -0.20000 | 0.00000 | 1.50000 |
| 14 | 260.00000 | -0.20000 | 0.00000 | 1.50000 |
| 15 | 280.00000 | -0.20000 | 0.00000 | 1.50000 |

| | | | | |
|---|---|---|---|---|
| 16 | 300.00000 | -0.20000 | 0.00000 | 1.00000 |
| 17 | 320.00000 | -0.20000 | 0.00000 | 3.00000 |
| 18 | 340.00000 | -0.20000 | 0.00000 | 1.00000 |

After several experimental runs of the program, we corrected these values and the final results are presented in Table 1.

The mechanism developed as a result of synthesis is demonstrated in Figure 1, its dimensions are given along with kinematic configuration. The crank length is $l_2 = AB = 0.372$.

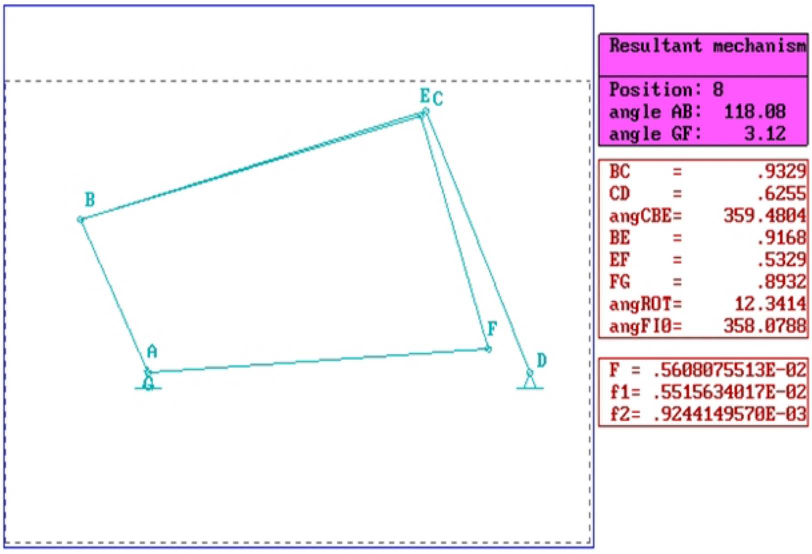


**Figure 1.** Kinematic configuration of nonharmonic vibration exciter

Below we report on the values of generalized eccentric masses, at which the regularity of the influence on the stand is achieved:

$A_2 = 9.604306E–02$ $B_2 = 5.042776E–02$

$A_4 = –1.252109E–01$ $B_4 = –8.328925E–02$

$A_6 = –1.886204+00$ $B_6 = –5.349329E+00$

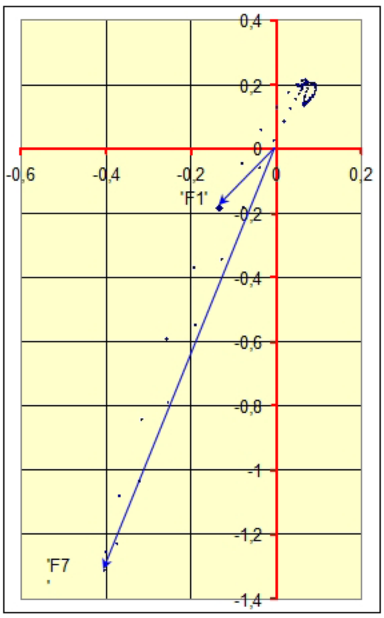


**Figure 2.** Hodograph of total inertia force for lever mechanism

The results of kinematic and kinetostatic analysis enable us to develop the graph of the force and the inertia acting on the foundation. In Figure 2, a hodograph describes the changes in the total impact action of inertia forces on foundation of the mechanism; in Table 1, the numerical values of components $F_x$ of $F_y$ are given.

**Research results**

Developed as the result of dynamic synthesis, the new mechanism of vibration exciter of directed pulse impact on foundation has advantages over the existing ones: the use of turning kinematic pairs allows elimination of the teeth wearing to the degree typical to its prototype; the regularity of inertia forces impact on the foundation is non-harmonic; the inertia forces acting in the orthogonal direction are actually absent.

On the base of hinge-lever type mechanisms, the program of dynamic synthesis of vibration exciter is developed. This permits faster completion of the designing cycle, gives opportunities to replace the expensive natural experiment with the computational one and enables multiple searches for solutions.

The analytic and optimization method is developed to determine optimal dimensions of the mechanism by employing the modified effectiveness function. As geometrical dimensions of the mechanism are included into

into effectiveness function nonlinearly (implicitly through angles of links turns, analogs of velocities and analogs of accelerations), the numerical optimization methods can be used to determine them. At this, it should be noted that the mass and inertia parameters of the mechanism are identified analytically on each step of descent to the minimum.

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