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The Partial Discretization Method of Differential Equations in Solving the Task on the Flexible Elastic Rotationally Symmetric Round Plate Flexure

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Abstract: The flexible elastic rotationally symmetric round plate with constant thickness *h*, which is being under the shearing distributed load along the circle, is examined. The obtaining of the analytical solution regarding the rotationally symmetric bending flexure of the round flexible elastic plate described by the non-linear system of differential equations is mathematically difficult. Thereupon the use of the method of partial discretization of differential equations, developed by one of the authors of the article, was found to be appropriate. At the same time in the class of aggregated functions the solution of the concerned task is obtained and the curve of the plate's bending flexure is depicted. The comparison of curves built using the relatively simple differential equations showed their almost full coincidence. Also the expressions of the rotation angle, stress, bending moments and discontinuous forces are found. The solutions for different rules on distribution of shearing load and boundary conditions are demonstrated. The obtained results are new in the theory of the bending flexure of the thin elastic plates.

Key words: Differential equations・Round flexible plate・Method of partial discretization・Determined

INTRODUCTION

Using the method of the partial discretization of differential equations [1] the analytical solution of the task on the flexible elastic rotationally symmetric round plate flexure, which is described by the non-linear system of differential equations [2, 3] is determined.

Examine the round flexible plate with a constant thickness *h*, which is being under the shearing distributed load with intensity $q(r)$.

The primary system of differential equations for the round flexible plate is as follows [1].

$$D\frac{d}{dr}\left(\nabla^2 w\right) = \Psi + \frac{h}{r}\frac{d\Phi}{dr}\frac{dw}{dr}$$
$$\frac{d}{dr}\left(\nabla^2 \Phi\right) = -\frac{E}{2r}\left(\frac{dw}{dr}\right)^2 \qquad (1)$$

where $\nabla^2 = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)$, $\Psi = \frac{1}{r}\int_0^r q_0\delta(r-r_c)rdr$ – load function, dispersed along the circle with a radius r_c, Φ–stress function, w- flexure, E–coefficient of elasticity, $D = \frac{Eh^3}{12\left(1-\mu^2\right)}$ – plate stiffness, μ– Poisson's ratio.

Solution Method: Using Tyurehodzhaev's method of partial discretization of differential equations [2] the second equation of the system (1) will look as follows

$$\frac{d^3\Phi}{dr^3} + \frac{1}{r}\frac{d^2\Phi}{dr^2} - \frac{1}{r^2}\frac{d\Phi}{dr} =$$

$$= -\frac{E}{4r}\sum_{k=1}^{n}\left(r_k + r_{k+1}\right)\left\{\left[\frac{dw(r_k)}{dr}\right]^2\delta\left(r-r_k\right) - \left[\frac{dw(r_{k+1})}{dr}\right]^2\delta\left(r-r_{k+1}\right)\right\} \qquad (2)$$

where $\delta(r - r_k)$ Dirak delta function. The general solution (2) is as follows

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$$\frac{d\Phi}{dr} = C_1 r + C_2 \frac{1}{r} - \frac{E}{8}\sum_{k=1}^{n}\left\{\left[\frac{dw(r_k)}{dr}\right]^2 (1+\frac{r_{k+1}}{r_k})(r-\frac{{r_k}^2}{r})H(r-r_k) - \right.$$

$$\left.\left[\frac{dw(r_{k+1})}{dr}\right]^2 (1+\frac{r_k}{r_{k+1}})(r-\frac{{r_{k+1}}^2}{r})H(r-r_{k+1})\right\}, \tag{3}$$

where $H(r-r_k)$ – Heaviside's single function; C_1, C_2– the constants of integration.

Placing the expression (3) in the first equation of the system (1) the discretization of the multiplier $\frac{dw}{dr}$ on the right side of the equation and the expression itself (3) will look as follows [3-7].

$$\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{d\ w}{dr} =$$

$$\frac{1}{D}\left[\Psi + \frac{h}{r}\left(C_1 r + C_2\frac{1}{r} - \frac{E}{8}\sum_{k=1}^{n}\left\{\left[\frac{dw(r_k)}{dr}\right]^2(1+\frac{r_{k+1}}{r_k})(r-\frac{{r_k}^2}{r})H(r-r_k) - \right.\right.\right.$$

$$\left.\left[\frac{dw(r_{k+1})}{dr}\right]^2(1+\frac{r_k}{r_{k+1}})(r-\frac{{r_{k+1}}^2}{r})H(r-r_{k+1})\right\}\sum_{k+1}^{n}\frac{1}{2}(r_k+r_{k+1})\left\{\left[\frac{dw(r_k)}{dr}\right]\delta(r-r_k) - \right.$$

$$\left.\left.\left.\left[\frac{dw(r_{k+1})}{dr}\right]\delta(r-r_{k+1})\right\}\right)\right] \tag{4}$$

Working on (4) taking into account the features of discontinuous functions on the right side of the equation the following expression for the rotation angle will be found:

$$\frac{dw}{dr} = C_3\frac{r}{2} + C_4\frac{1}{r} + \frac{1}{D}\left[\frac{1}{r}\int\left(r\int\psi(r)\,dr\right)dr + \frac{hr}{4}\left\{(r_1+r_2)\left(C_1 + C_2\frac{1}{r_1^2}\right)\left(1-\frac{r_1^2}{r_2}\right)\cdot\right.\right.$$

$$\left.\left[\frac{dw(r_1)}{dr}\right]H(r-r_1) + \sum_{i=2}^{n}(r_{i+1}-r_{i-1})(C_1 + C_2\frac{1}{r_i^2})(1-\frac{r_i^2}{r^2})\left[\frac{dw(r_i)}{dr}\right]H(r-r_i)\right\} -$$

$$\frac{Ehr}{32}\left(1+\frac{r_2}{r_1}\right)\sum_{k=2}^{n}(r_{k+1}-r_{k-1})\left\{\left(1-\frac{r_1^2}{r_k^2}\right)\times(1-\frac{r_k^2}{r^2})\left[\frac{dw(r_1)}{dr}\right]^2 + \sum_{j=2}^{k-1}\frac{(r_{j+1}-r_{j-1})}{r_j}\cdot\right.$$

$$\left.\left.\left(1-\frac{r_j^2}{r_k^2}\right)\left(1-\frac{r_k^2}{r^2}\right)\left[\frac{dw(r_k)}{dr}\right]^2\right\}\left[\frac{dw(r_k)}{dr}\right]H(r-r_k)\right] \tag{5}$$

Taking into account the features of the stress function $\frac{d\Phi}{dr}$ and the rotation angle $\frac{dw}{dr}$ in the point $r = 0$, we get $C_2 = C_4 = 0$. Then

$$\frac{d\Phi}{dr} = C_1 r - \frac{Er}{8}\left\{\left(1+\frac{r_2}{r_1}\right)\left(1-\frac{r_1^2}{r^2}\right)\left[\frac{dw(r_1)}{dr}\right]^2 H(r-r_1) + \right.$$

$$\left. \sum_{k=2}^{n}\frac{(r_{k+1}-r_{k-1})}{r_k}(1-\frac{r_k{}^2}{r^2})\left[\frac{dw(r_k)}{dr}\right]^2 H(r-r_k)\right\} \tag{6}$$

$$\frac{dw}{dr} = C_3\frac{r}{2} + \frac{q_0(r_b{}^2-r_a{}^2)^2}{4Dr}(\ln\frac{r_b}{r_a}-\frac{1}{2}) + \frac{hC_1 r}{4}\left\{(r_1+r_2)\left(1-\frac{r_1^2}{r^2}\right)\left[\frac{dw(r_1)}{dr}\right]H(r-r_1) + \right.$$

$$\left. \sum_{i=2}^{n}(r_{i+1}-r_{i-1})\times\left(1-\frac{r_i^2}{r^2}\right)\left[\frac{dw(r_i)}{dr}\right]H(r-r_i)\right\} - \frac{Ehr}{32}(1+\frac{r_2}{r_1})\sum_{k=2}^{n}(r_{k+1}-r_{k-1})\cdot$$

$$\left\{\left(1-\frac{r_1^2}{r_k^2}\right)\left(1-\frac{r_k^2}{r^2}\right)\left[\frac{dw(r_1)}{dr}\right]^2 + \sum_{j=2}^{k-1}\frac{(r_{j+1}-r_{j-1})}{r_j}(1-\frac{r_j{}^2}{r_k{}^2})(1-\frac{r_k{}^2}{r^2})\left[\frac{dw(r_j)}{dr}\right]^2\right\}\cdot$$

$$\left[\frac{dw(r_k)}{dr}\right]H(r-r_k) \tag{7}$$

Assuming that the ring plate with an inward radius r_a and an outward radius r_b is leaned on the secondary duct and on the main duct the constant intensity stress σ_0 is applied the boundary conditions will look as follows [8-11].

$$\sigma_r(r) = \frac{1}{r}\frac{d\Phi}{dr} = \sigma_0, \quad r = r_a. \tag{8}$$

$$M_r(r) = -D(\frac{d^2w}{d^2r} + \frac{\mu}{r}\frac{dw}{dr}) = 0, \quad r = r_b. \tag{9}$$

$$w(r) = 0, \quad r = r_b. \tag{10}$$

The use of the boundary conditions (8) and (9) gives

$$\frac{dw}{dr} = -\frac{q_0 r_c r}{4D}\left\{2\left[\ln\frac{r_b}{r_a} - \ln\frac{r}{r_c}H(r-r_c)\right] + \left[\frac{(1-\mu)}{(1+\mu)}\left(1-\frac{r_c^2}{r_b^2}\right) + \left(1-\frac{r_c^2}{r^2}\right)\cdot\right.\right.$$

$$H(r-r_c)]\} - \frac{h\sigma_0 r}{4D(1+\mu)}\left((r_1+r_2)\left\{\left[\left(1+\frac{r_1^2}{r_b{}^2}\right)+\mu\left(1-\frac{r_1^2}{r_b{}^2}\right)\right]-\right.\right.$$

$$(1+\mu)\times\left(1-\frac{r_1^2}{r^2}\right)H(r-r_1)\bigg\}\left[\frac{dw(r_1)}{dr}\right] + \sum_{i=2}^{n}(r_{i+1}-r_{i-1})\left\{\left[\left(1+\frac{r_i^2}{r_b^2}\right)+\mu\left(1-\frac{r_i^2}{r_b^2}\right)\right]-(1+\mu)\cdot\right.$$

$$\left(1-\frac{r_i^2}{r^2}\right)H(r-r_i)\bigg\}\times\left[\frac{dw(r_i)}{dr}\right]\Bigg\rangle + \frac{Ehr}{32D(1+\mu)}\left(1+\frac{r_2}{r_1}\right)\sum_{k=2}^{n}(r_{k+1}-r_{k-1})\left(1-\frac{r_1^2}{r_k^2}\right)\Bigg\langle\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\right.$$

$$\mu\left(1-\frac{r_k^2}{r_b^2}\right)\Bigg]-(1+\mu)\times\left(1-\frac{r_k^2}{r^2}\right)H\left(r-r_k\right)\Bigg\}\left[\frac{dw\left(r_1\right)}{dr}\right]^2+\sum_{j=2}^{k-1}\frac{\left(r_{j+1}-r_{j-1}\right)}{r_j}\left(1-\frac{r_j^2}{r_k^2}\right)\cdot$$

$$\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-(1+\mu)\times\left(1-\frac{r_k^2}{r^2}\right)H\left(r-r_k\right)\right\}\left[\frac{dw(r_j)}{dr}\right]^2\Bigg)\left[\frac{dw(r_k)}{dr}\right]. \tag{11}$$

Taking into consideration (10) the final expression of the plate flexure $w(r)$, stress $\sigma_r(r)$ and $\sigma_\theta(r)$, bending moment's $M_r(r)$ и $M_\theta(r)$ will look as follows

$$w\left(r\right)=-\frac{q_0r_cr^2}{4D}\left\langle\left[\ln\frac{r_b}{r_c}+\frac{(1-\mu)}{2(1+\mu)}\left(1-\frac{r_c^2}{r_b^2}\right)\right]\left[\left(1-\frac{r_c^2}{r^2}\right)-\frac{r_b^2}{r^2}\cdot\right.\right.$$

$$\left.\left(1-\frac{r_c^2}{r_b^2}\right)\right]+\left\{\left[(1-\frac{r_c^{\,2}}{r^2})-\ln\frac{r}{r_c}(1+\frac{r_c^{\,2}}{r^2})\right]H(r-r_c)-\left[\left(1-\frac{r_c^{\,2}}{r_b^{\,2}}\right)-\ln\frac{r_b}{r_c}\left(1+\frac{r_c^{\,2}}{r_b^{\,2}}\right)\right]\right\}\Bigg\rangle-$$

$$\frac{h\sigma_0r^2}{8D(1+\mu)}\times\left[(r_1+r_2)\left(\left\{\left[\left(1+\frac{r_1^2}{r_b^2}\right)+\mu\left(1-\frac{r_1^2}{r_b^2}\right)\right]-2(1+\mu)\frac{r_1^2}{r^2}\left[\frac{1}{2}\left(\frac{r^2}{r_1^2}-1\right)-\ln\frac{r}{r_1}\right]\cdot\right.\right.\right.$$

$$H\left(r-r_1\right)\}-\frac{r_b^2}{r^2}\left\{\left[\left(1+\frac{r_1^2}{r_b^2}\right)+\mu\left(1-\frac{r_1^2}{r_b^2}\right)\right]-2(1+\mu)\frac{r_1^2}{r_b^2}\left[\frac{1}{2}\left(\frac{r_b^2}{r_1^2}-1\right)-\ln\frac{r_b}{r_1}\right]\right\}\Bigg)\left[\frac{dw\left(r_1\right)}{dr}\right]+$$

$$\sum_{i=2}^{n}\left(r_{i+1}-r_{i-1}\right)\left(\left\{\left[\left(1+\frac{r_i^2}{r_b^2}\right)+\mu\left(1-\frac{r_i^2}{r_b^2}\right)\right]-2(1+\mu)\frac{r_i^2}{r^2}\left[\frac{1}{2}\left(\frac{r^2}{r_i^2}-1\right)-\ln\frac{r}{r_i}\right]\cdot\right.\right.$$

$$H\left(r-r_i\right)\}-\frac{r_b^2}{r^2}\left\{\left[\left(1+\frac{r_i^2}{r_b^2}\right)+\mu\left(1-\frac{r_i^2}{r_b^2}\right)\right]-2(1+\mu)\frac{r_i^2}{r_b^2}\left[\frac{1}{2}\left(\frac{r_b^2}{r_i^2}-1\right)-\ln\frac{r_b}{r_i}\right]\right\}\Bigg)\left[\frac{dw(r_i)}{dr}\right]\Bigg]+$$

$$\frac{Ehr^2}{64D(1+\mu)}\left(1+\frac{r_2}{r_1}\right)\sum_{k=2}^{n}(r_{k+1}-r_{k-1})\left(1-\frac{r_1^2}{r_k^2}\right)\left[\left(\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-\right.\right.\right.$$

$$2(1+\mu)\frac{r_k^2}{r^2}\left[\frac{1}{2}\left(\frac{r^2}{r_k^2}-1\right)-\ln\frac{r_b}{r_k}\right]H\left(r-r_k\right)\Bigg\}-\frac{r_b^2}{r^2}\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-\right.$$

$$2(1+\mu)\frac{r_k^2}{r_b^2}\left[\frac{1}{2}\left(\frac{r_b^2}{r_k^2}-1\right)-\ln\frac{r_b}{r_k}\right]\Bigg\}\Bigg)\left[\frac{dw\left(r_1\right)}{dr}\right]^2+\sum_{j=2}^{k-1}\frac{(r_{j+1}-r_{j-1})}{r_j}\left(1-\frac{r_j^{\,2}}{r_k^{\,2}}\right)\left(\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\right.\right.\right.$$

$$\mu\left(1-\frac{r_k^2}{r_b^2}\right)\Bigg]-2(1+\mu)\frac{r_k^{\,2}}{r^2}\left[\frac{1}{2}\left(\frac{r^2}{r_k^2}-1\right)-\ln\frac{r}{r_k}\right]H\left(r-r_k\right)\Bigg\}-\frac{r_b^{\,2}}{r^2}\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\right.\right.$$

$$\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-2\left(1+\mu\right)\frac{r_k{}^2}{r_b^2}\left[\frac{1}{2}\left(\frac{r_b^2}{r_k^2}-1\right)-\ln\frac{r_b}{r_k}\right]\right\}\right)\left[\frac{dw\left(r_j\right)}{dr}\right]^2\right]\left[\frac{dw\left(r_k\right)}{dr}\right] \quad (12)$$

$$\sigma_r\left(r\right)=\sigma_0-\frac{E}{8}\left\{\left(1+\frac{r_2}{r_1}\right)\left(1-\frac{r_1^2}{r^2}\right)\left[\frac{dw\left(r_1\right)}{dr}\right]^2 H\left(r-r_1\right)+\right.$$

$$\left.\sum_{k=2}^{n}\frac{(r_{k+1}-r_{k-1})}{r_k}\left(1-\frac{r_k^2}{r^2}\right)\left[\frac{dw\left(r_k\right)}{dr}\right]^2 H\left(r-r_k\right)\right\} \quad (13)$$

$$\sigma_\theta\left(r\right)=\sigma_0-\frac{E}{8}\left\{\left(1+\frac{r_2}{r_1}\right)\left(1+\frac{r_1^2}{r^2}\right)\left[\frac{dw\left(r_1\right)}{dr}\right]^2 H\left(r-r_1\right)+\right.$$

$$\left.\sum_{k=2}^{n}\frac{(r_{k+1}-r_{k-1})}{r_k}\left(1+\frac{r_k^2}{r^2}\right)\left[\frac{dw\left(r_k\right)}{dr}\right]^2 H\left(r-r_k\right)\right\}; \quad (14)$$

$$M_r\left(r\right)=\frac{q_0 r_c}{4}\left\langle 2\left\{\left[\ln\frac{r_b}{r_c}-(\ln\frac{r}{r_c}+1)H(r-r_c)\right]+\mu\left[\ln\frac{r_b}{r_c}-\ln\frac{r}{r_c}H(r-r_c)\right]\right\}+\left\{\left[\frac{(1-\mu)}{(1+\mu)}(1-\frac{r_c{}^2}{r_b{}^2})+\right.\right.\right.$$

$$\left.\left.\left.(1+\frac{r_c{}^2}{r^2})H(r-r_c)\right]+\mu\left[\frac{(1-\mu)}{(1+\mu)}(1-\frac{r_c{}^2}{r_b{}^2})+(1-\frac{r_c{}^2}{r^2})H(r-r_c)\right]\right\}\right\rangle+\frac{h\sigma_0}{4\left(1+\mu\right)}\left[\left(r_1+r_2\right)\left(\left\{\left[\left(1+\frac{r_1^2}{r_b^2}\right)+\right.\right.\right.\right.$$

$$\left.\mu\left(1-\frac{r_1^2}{r_b^2}\right)\right]-\left(1+\mu\right)\left(1+\frac{r_1^2}{r^2}\right)H\left(r-r_1\right)\right\}+\mu\left\{\left[\left(1+\frac{r_1^2}{r_b^2}\right)+\mu\left(1-\frac{r_1^2}{r_b^2}\right)\right]-\left(1+\mu\right)\left(1-\frac{r_1^2}{r^2}\right)\cdot\right.$$

$$\left.H\left(r-r_1\right)\right\}\right)\left[\frac{dw\left(r_1\right)}{dr}\right]+\sum_{i=2}^{n}\left(r_{i+1}-r_{i-1}\right)\left(\left\{\left[\left(1+\frac{r_i{}^2}{r_b{}^2}\right)+\mu\left(1-\frac{r_i{}^2}{r_b{}^2}\right)\right]-\left(1+\mu\right)\left(1+\frac{r_i{}^2}{r^2}\right)H\left(r-r_i\right)\right\}+\right.$$

$$\left.\mu\left\{\left[\left(1+\frac{r_i^2}{r_b^2}\right)+\mu\left(1-\frac{r_i^2}{r_b^2}\right)\right]-\left(1+\mu\right)\left(1-\frac{r_i^2}{r^2}\right)H\left(r-r_i\right)\right\}\right)\left[\frac{dw\left(r_i\right)}{dr}\right]-\frac{Eh}{32\left(1+\mu\right)}\left(1+\frac{r_2}{r_1}\right)\cdot$$

$$\sum_{k=2}^{n}\left(r_{k+1}-r_{k-1}\right)\left(1-\frac{r_1{}^2}{r_k{}^2}\right)\left[\left(\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-\left(1+\mu\right)\left(1+\frac{r_k^2}{r^2}\right)H\left(r-r_k\right)\right\}+\mu\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\right.\right.\right.$$

$$\left.\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-\left(1+\mu\right)\left(1-\frac{r_k^2}{r^2}\right)H\left(r-r_k\right)\right\}\right)\left[\frac{dw\left(r_1\right)}{dr}\right]^2+\sum_{j=2}^{k-1}\frac{\left(r_{j+1}-r_{j-1}\right)}{r_j}\left(1-\frac{r_j^2}{r_k^2}\right)\left(\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\mu\left(1-\frac{r_{rk}^2}{r_b^2}\right)\right]-\right.\right.$$

$$\left(1+\mu\right)\times\left(1+\frac{r_k{}^2}{r^2}\right)H\left(r-r_k\right)\right\}+\mu\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-\right.$$

$$\left(1+\mu\right)\left(1-\frac{r_k^2}{r^2}\right)H\left(r-r_k\right)\Bigg\}\Bigg)\left[\frac{dw\left(r_j\right)}{dr}\right]^2\Bigg]\left[\frac{dw\left(r_k\right)}{dr}\right] \tag{15}$$

$$M_\theta\left(r\right)=\frac{q_0r_c}{4}\Bigg\langle 2\left\{\left[\ln\frac{r_b}{r_c}-\ln\frac{r}{r_c}H(r-r_c)\right]+\mu\left[\ln\frac{r_b}{r_c}-(\ln\frac{r}{r_c}+1)H(r-r_c)\right]\right\}+$$

$$\left\{\left[\frac{(1-\mu)}{(1+\mu)}\left(1-\frac{r_c^2}{r_b^2}\right)+(1-\frac{r_c{}^2}{r^2})H(r-r_c)\right]+\mu\left[\frac{(1-\mu)}{(1+\mu)}\left(1-\frac{r_c^2}{r_b^2}\right)+\right.\right.$$

$$\left.\left.\left(1+\frac{r_c^2}{r^2}\right)H\left(r-r_c\right)\right]\right\}\Bigg\rangle+\frac{h\sigma_0}{4\left(1+\mu\right)}\Big[\left(r_1+r_2\right)\cdot$$

$$\Bigg(\left\{\left[\left(1+\frac{r_1^2}{r_b^2}\right)+\mu\left(1-\frac{r_1^2}{r_b^2}\right)\right]-\left(1+\mu\right)\left(1-\frac{r_1^2}{r^2}\right)H\left(r-r_1\right)\right\}+\mu\left\{\left[\left(1+\frac{r_1^2}{r_b^2}\right)+\mu\left(1-\frac{r_1^2}{r_b^2}\right)\right]-\left(1+\mu\right)\cdot\right.$$

$$\left.\left(1+\frac{r_1^2}{r^2}\right)H\left(r-r_1\right)\right\}\Bigg)\left[\frac{dw(r_1)}{dr}\right]+\sum_{i=2}^{n}(r_{i+1}-r_{i-1})\Bigg(\left\{\left[\left(1+\frac{r_i^2}{r_b^2}\right)+\mu\left(1-\frac{r_i^2}{r_b^2}\right)\right]-\left(1+\mu\right)\cdot\right.$$

$$\left.\left(1-\frac{r_i^2}{r^2}\right)H\left(r-r_i\right)\right\}+\mu\left\{\left[\left(1+\frac{r_i^2}{r_b^2}\right)+\mu\left(1-\frac{r_i^2}{r_b^2}\right)\right]-\left(1+\mu\right)\left(1+\frac{r_i^2}{r^2}\right)H\left(r-r_i\right)\right\}\Bigg)\cdot$$

$$\left[\frac{dw(r_i)}{dr}\right]-\frac{Eh}{32\left(1+\mu\right)}\left(1+\frac{r_2}{r_1}\right)\sum_{k=2}^{n}(r_{k+1}-r_{k-1})\left(1-\frac{r_1^2}{r_k^2}\right)\Bigg[\Bigg(\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\right.\right.$$

$$\left.\left.\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-\left(1+\mu\right)\left(1-\frac{r_k^2}{r^2}\right)H\left(r-r_k\right)\right\}+\mu\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-\right.$$

$$\left.\left(1+\mu\right)\left(1+\frac{r_k^2}{r^2}\right)H\left(r-r_k\right)\right\}\Bigg)\left[\frac{dw(r_i)}{dr}\right]^2+\sum_{j=2}^{k-1}\frac{(r_{j+1}-r_{j-1})}{r_j}\left(1-\frac{r_j{}^2}{r_k{}^2}\right)\Bigg(\left\{\left[\left(1+\frac{r_k{}^2}{r_b^2}\right)+\mu\left(1-\frac{r_k{}^2}{r_b^2}\right)\right]-\right.$$

$$\left.\left(1+\mu\right)\times\left(1-\frac{r_k{}^2}{r^2}\right)H\left(r-r_k\right)\right\}+\mu\left\{\left[\left(1+\frac{r_k^2}{r_b^2}\right)+\mu\left(1-\frac{r_k^2}{r_b^2}\right)\right]-\right.$$

$$\left.\left(1+\mu\right)\left(1+\frac{r_k^2}{r^2}\right)H\left(r-r_k\right)\right\}\Bigg)\left[\frac{dw\left(r_j\right)}{dr}\right]^2\Bigg]\left[\frac{dw\left(r_k\right)}{dr}\right] \tag{16}$$

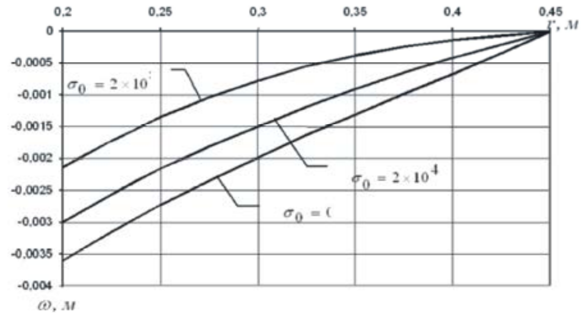

Fig. 1: Results decreasing the radial load σ_0.

RESULTS AND DISCUSSION

On the picture the integral curve (1), (8), (9), (10), determined by five points at

$h = 0{,}02м$; $q_0 = 2{,}5 \cdot 10^5 \frac{H}{m^2}$; $\sigma_0 = 0$, $\sigma_0 = 2 \cdot 10^4 \frac{H}{m^2}$, $\sigma_0 = 2 \cdot 10^8 \frac{H}{m^2}$; $E = 2 \cdot 10^{11} \frac{H}{m^2}$; $\mu = 0{,}3$ is depicted.

As appears form the above, decreasing the radial load σ_0 on the main duct the blending flexure of the plate is increasing.

In this paper the solutions for different rules on distribution of shearing load and boundary conditions are demonstrated.

CONCLUSION

Although the existing method of Bubnov-Galerkin for the solution of non-linear differential equations is useful however in general formulation the linear combination of the specified linearly independent system cannot assure even weak convergence of the approximate solution to the precise one. Thereupon the appliance of the partial discretization method to the concerned system of non-linear equations is found to be appropriate.

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